BUS211 – Financial Mathematics

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Chapter 1: Introduction to basic mathematics and Simple Interest

Learning Objectives

- Review and Applications of Basic Mathematics
- Review and Applications of Algebra
- Ratios and Proportions
- Linear equations
- **Simple Interest**

LO1: Arithmetic Operations

- Algebraic Operating System (AOS) – the rules that we use to evaluate an expression (also know as BEDMAS or Order of Operations).

1. Perform all operations within brackets.
2. Evaluate the powers (exponents).
3. Perform multiplication and division.
4. Perform addition and subtraction.

- BEDMAS - Brackets, Exponents, Division, Multiplication, Addition, Subtraction

Order of Operations Example

- Consider the following expression.
- BEDMAS tells us to complete the division first and then the addition/subtractions.

$$30 - 6 \div 3 + 5$$

$$= 30 - 2 + 5$$

$$= 33$$

Order of Operations Example

- Consider the following expression.
- BEDMAS tells us to complete the bracket first, then the exponent and then the addition/subtractions.

$$10 + (2 \times 3)^2 - 5$$

$$= 10 + 6^2 - 5$$

$$= 10 + 36 - 5$$

$$= 41$$

Fractions, Decimals & Percents

- A faction consists of a numerator and a denominator.

$$\frac{numerator}{denominator}$$

- Proper faction – the numerator is smaller than the denominator.

$$\frac{3}{4}$$

- Improper faction – the numerator is larger than the denominator.

$$\frac{5}{2}$$

- Mixed number – a whole number plus a fraction.

$$2\frac{1}{2}$$

Decimal and Percent Equivalents

- The decimal equivalent of a fraction is obtained by dividing the numerator by the denominator.
- The percent equivalent is found by multiplying the decimal equivalent by 100 and adding the % sign.

$$\frac{3}{4} = 0.75 = 75\%$$

Fraction — Decimal Equivalent — Percent Equivalent

LO4: Calculating Percent of a Number

- To find the percent of a number, convert the percent to its decimal equivalent by diving the percent by 100 and then multiplying the number.

- For example:

$$22\%\ of\ \$185\ becomes\ 0.22 \times \$185 = \$40.70$$

Example 1.2J – A Problem Using Percents

A battery manufacturer encloses a rebate coupon for 15% off in a package of two AAA batteries retailing for \$6.29. What rebate does the coupon represent?

Solution: Convert the percentage to a decimal and then find the rebate.

$$Rebate = 0.15 \times \$6.29 = \$0.94$$

The 15% rebate is equivalent to a cash rebate of \$0.94.

LO6: Simple and Weighted Averages

- Determining the average of a set of numbers is a useful business calculation.
- A simple average is the sum of a set of values divided by the number of values in the set.
  - It is useful in cases where each item in the set has the same importance.
- A weighted average attaches a weighting factor to each value to represent its relative importance.
  - This is useful in cases where each item in the set has a different level of importance.

Simple Averages

- To calculate a simple average, we take the sum of a set of values and divide by the number of values in the set.

$$\text{Simple average} = \frac{\text{Sum of the values}}{\text{Total number of values}}$$

- For example, the simple average age of three people ages 20, 23 and 24 would be:

$$\frac{20 + 23 + 24}{3} = 22.3\overline{3}$$

Weighted Averages

- A weighted average includes the relative importance, or "weight", of each item in the set.

- For example, if a company has 5 employees in total and 2 of them earn $12.00/hr while the other 3 earn $15.00/hr, the weighted average would be calculated as follows:

$$\frac{2 \times \$12.00 + 3 \times \$15.00}{5} = \$13.80$$

Review and Applications of Algebra

LO1: Algebraic Operations

- Algebraic expression – indicates the mathematical operations to be carried out on a combination of numbers and variables.

- Terms – the components of an algebraic expression that are separated by addition or subtraction signs.

$$\$2500 + 0.04s$$

Algebraic Operations

Monomial – an expression containing 1 term.

$3x^2$

Binomial – an expression containing 2 terms.

$3x^2 + xy$

Trinomial – an expression containing 3 terms.

$3x^2 + xy - 6y^2$

Polynomial – any expression containing more than 1 term.

Addition and Subtraction

- Algebraic expressions may be simplified by adding or subtracting like terms.
- Also called combining like terms.

$$3a - 4b - 7a + 9b$$

$$3a - 7a - 4b + 9b$$

$$-4a + 5b$$

Multiplication and Division

- To multiply two polynomials, multiply each term in one polynomial by each term in the other.
- Also called the distributive property.

$$(a+b)(c+d+e)$$

$$a(c+d+e)+b(c+d+e)$$

$$ac+ad+ae+bc+bd+be$$

Multiplication and Division

- Don't forget, after you multiply the two polynomials, you may be able to collect like terms.

$$(7a - 2b)(3b - 2a)$$

$$7a(3b - 2a) - 2b(3b - 2a)$$

$$21ab - 14a^2 - 6b^2 + 4ab$$

$$25ab - 14a^2 - 6b^2$$

Multiplication and Division

- To divide by a monomial, divide each term in the numerator by the denominator.

$$\frac{48a^2 - 32ab}{8a}$$

$$\frac{48a^2}{8a} - \frac{32ab}{8a}$$

$$6a - 4b$$

Rules of Exponents

Rule	Example
$a^m \times a^n = a^{m+n}$	$2^2 \times 2^3 = 2^5 = 32$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = 2^1 = 2$
$(a^m)^n = a^{m \times n}$	$(2^2)^3 = 2^6 = 64$
$(ab)^n = a^n b^n$	$(5x)^3 = 5^3 x^3 = 125x^3$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{x}{3}\right)^2 = \frac{x^2}{3^2} = \frac{x^2}{9}$

Rules of Exponents

Rule	Example
$a^0 = 1$	$5^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$5^{-2} = \frac{1}{5^2} = \frac{1}{25} = 0.04$
$a^{1/n} = \sqrt[n]{a}$	$9^{1/2} = \sqrt[2]{9} = 3$
$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$	$6^{3/2} = (\sqrt[2]{6})^3 = \sqrt[2]{6^3}$ $= \sqrt[2]{216} = 14.69694$

A General Approach for Solving Word Problems

1. Read the entire problem.
2. Extract and label the data. Identify the unknown quantity and specify its symbol. Draw and label a diagram if appropriate.
3. Create a word equation that relates the given data to the unknown quantity.
4. Convert the word equation to an algebraic equation.
5. Solve the equation.

Example 2.5B – A Problem Using Percents

A retailer reduced prices by 15% for a fall sale. What was the regular price of an item on sale for $123.25?

Step 2: Extract and label the data. Identify the unknown and define its symbol.

Discount rate = 15% Sale price = $123.25 Let P = the regular price.

Step 3: Create a word equation that relates the given data to the unknown quantity.

Sale price = Regular price – Price reduction

= Regular price – (Discount rate × Regular price)

Step 4: Convert the word equation to an algebraic equation.

$123.25 = P – 0.15P

Step 5: Solve the equation.

$123.25 = 0.85P

P = $145.00

LO5: The Basic Percentage Problem

- The basic percentage formula is:

$$Rate = \frac{Portion}{Base}$$

- Given any two of the three quantities, we can calculate the third.

Example: The Basic Percentage Problem

What is 25% of $63.00?

$$Rate = \frac{Portion}{Base}$$

$$0.25 = \frac{Portion}{\$63.00}$$

$$0.25 \times \$63.00 = Portion$$

$$\boldsymbol{Portion = \$15.75}$$

Example: The Basic Percentage Problem

$40 is 16% of what amount?

$$Rate = \frac{Portion}{Base}$$

$$0.16 = \frac{\$40.00}{Base}$$

$$Base = \frac{\$40.00}{0.16}$$

$$\boldsymbol{Base = \$250.00}$$

LO6: Percent Change

- When a quantity changes, the amount of change is often expressed as a percentage of the initial value.

$$c = \frac{V_f - V_i}{V_i} \times 100\%$$

V_i = Initial (or beginning or original) value
V_f = Final (or ending or new) value
c = Percent change

Example 2.7A – Calculating the Percent Change

The minimum wage in Alberta in 2009 was $8.80 per hour. This increased to $11.20 per hour in 2015. What is the percent change in the minimum wage for this time period?

$$c = \frac{2015\ wage\ - 2009\ wage}{2009\ wage} \times 100\%$$

$$c = \frac{\$11.20 - \$8.80}{\$8.80} \times 100\%$$

$$c = \frac{\$2.40}{\$8.80} \times 100\%$$

$$\boldsymbol{c = 27.27\%}$$

Ratios and Proportions

LO1: Ratios

- A ratio is a comparison of two or more quantities.
- Ratios may be expressed using a colon, as a fraction, as a decimal or as a percent.

$$5:10 \qquad \frac{5}{10} \qquad 0.5 \qquad 50\%$$

- A ratio can have more than 2 terms.

$$3:8:4$$

LO1: Ratios

- A ratio can be reduced to its lowest terms.
  1. If all terms are integers, divide every term by the common factors of all terms.
  2. If one or more terms are decimal numbers, make all the terms integers by moving the decimal point and then reduce as above.
  3. If one or more terms contain a fraction, the fractions should first be resolved and then reduced as above.

Example: Reducing Ratios

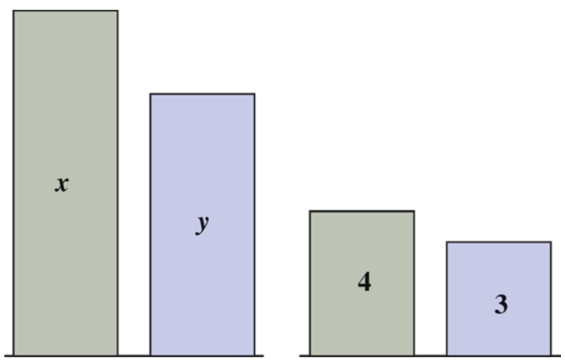
The costs of manufacturing an item are $225 for labour, $150 for materials, and $75 for overhead expenses. Express the ratio of labour cost to materials cost to overhead expenses in lowest terms.

225:150:75	Ratio with the given terms
9:6:3	Equivalent ratio (each term divided by 25)
3:2:1	Equivalent ratio with lowest terms (each term divided by 3)

Proportions

- A proportion is a statement of the equality of two ratios.
- Consider the statement "the ratio of the sales of X to the sales of Y is 4:3"
- This can be expressed as: $x : y = 4 : 3$

- Graphically, the columns x and y are proportional to 4 and 3.



Proportions

- We can use proportions to solve problems.
- First, convert the ratio to its equivalent fraction.

$$\frac{x}{y} = \frac{4}{3}$$

- Then, given either x or y, you can solve for the other variable.

Proportions

- Let's solve for y when x is $1800:

$$\frac{x}{y} = \frac{4}{3}$$

$$\frac{\$1800}{y} = \frac{4}{3}$$

$$\frac{\$1800}{y} \times 3y = \frac{4}{3} \times 3y$$

$$\$1800 \times 3 = 4y$$

$$y = \$1350$$

Example:

Betty and Lois have already invested $8960 and $6880, respectively, in their partnership. If Betty invests another $5000, what amount should Lois contribute to maintain their investments in the original ratio?

Lois's investment : Betty's investment = $6880 : 8960

Let Lois's additional investment be x. Then

$$\frac{x}{\$5000} = \frac{\$6880}{\$8960}$$

Therefore,

$$x = \frac{\$6880 \times \$5000}{\$8960} = \$3839.29$$

Allocation and Proration

- Often, money must be allocated among partners, departments, cost centers, etc.
- If the allocation is not made equally, we can use a procedure called proration.
- Proration allows us to allocate money on a proportionate basis.

Example:

The partnership of Mr. X, Mr. Y, and Ms. Z has agreed to distribute profits in the same proportion as their respective capital investments in the partnership. How will the recent period's profit of $28,780 be allocated if Mr. X's capital account shows a balance of $34,000, Mr. Y's shows $49,000, and Ms. Z's shows $54,500?

The total amount invested by all three partners = $34,000 + $49,000 + $54,500 = $137,500

We can use this ratio to determine the amount each partner will receive:

$$\frac{Partner's\ share}{Total\ profit} = \frac{Partner's\ investment}{Total\ investment}$$

Mr. X: $\frac{Mr.X's\ share}{\$28{,}780} = \frac{\$34{,}000}{\$137{,}500} \rightarrow Mr.X's\ share = \frac{\$34{,}000}{\$137{,}500} \times \$28{,}780 = \$7{,}116.51$

Mr. Y: $\frac{Mr.Y's\ share}{\$28{,}780} = \frac{\$49{,}000}{\$137{,}500} \rightarrow Mr.X's\ share = \frac{\$49{,}000}{\$137{,}500} \times \$28{,}780 = \$10{,}256.15$

Mr. Z: $\frac{Mr.Z's\ share}{\$28{,}780} = \frac{\$54{,}500}{\$137{,}500} \rightarrow Mr.X's\ share = \frac{\$54{,}500}{\$137{,}500} \times \$28{,}780 = \$11{,}407.35$

Linear equations

Graphical Method for Solving Two Equations in Two Unknowns

- If we plot two equations on the same graph, we can see the point at which they cross.
- This is the pair of (x,y) values that satisfy both equations.
- In other words, this is the solution to both equations.

Graphical Method for Solving Two Equations in Two Unknowns

Consider the two equations:

$$x - 2y = -2$$
$$x + y = 4$$

We can re-arrange them into the slope-intercept form as follows:

$$y = 1 + 0.5x$$
$$y = 4 - x$$

Graphical Method for Solving Two Equations in Two Unknowns

$$y = 1 + 0.5x$$
$$y = 4 - x$$

For each equation, calculate y when x=-4 and x=4:

$y = 1 + 0.5x$

x:	−4	4
y:	−1	3

$y = 4 - x$

x:	−4	4
y:	8	0

Graphical Method for Solving Two Equations in Two Unknowns

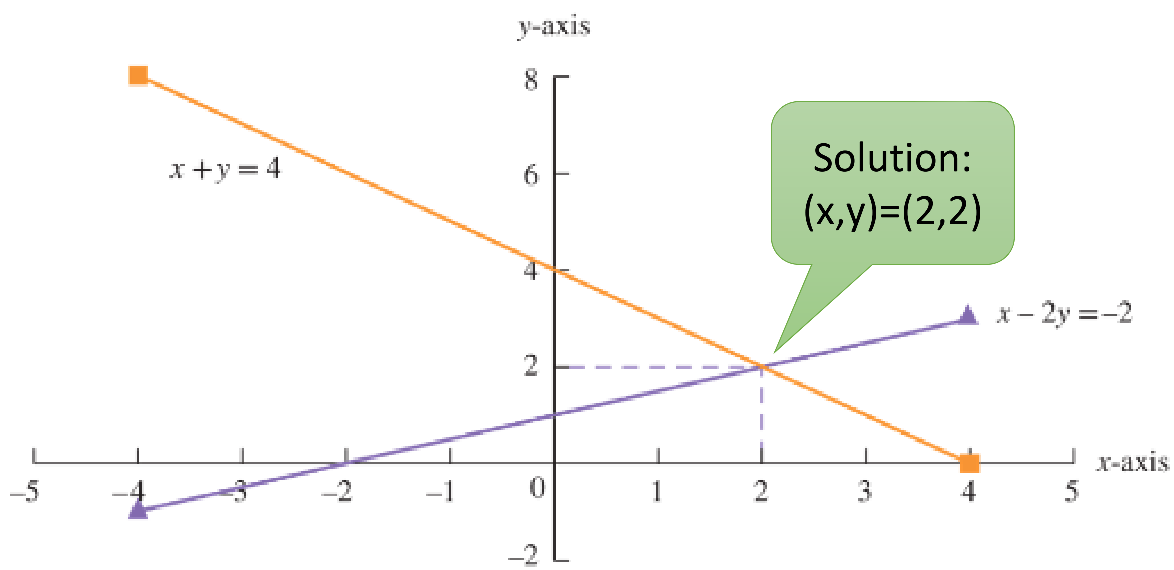
Graph each equation and determine the solution:

$y = 1 + 0.5x$

x:	−4	4
y:	−1	3

$y = 4 - x$

x:	−4	4
y:	8	0



Solving Two Equations in Two Unknowns Algebraically

- The previous example showed how to find the one set of values that satisfied both equations at the same time using a graph.

- We can also do this algebraically using:
  - the substitution method
  - the elimination method

Example: Substitution Method

Solve the following pair of equations.

$$y = 4x + 7 \quad ① \qquad 3x + y = 14 \quad ②$$

We know from equation ① that $y = 4x + 7$. We can substitute that into equation ② to get:

$$3x + (4x + 7) = 14$$

Now we only have one unknown (x) so we can solve for it:

$$3x + (4x + 7) = 14$$
$$7x = 7$$
$$x = 1$$

Substitute $x = 1$ back into equation ① to get:

$$y = 4(1) + 7$$
$$y = 11$$

So, the solution is $(x, y) = (1,11)$

Example: Elimination Method

Solve the following pair of equations.

$$2x - 3y = -6 \quad ① \qquad x + y = 2 \quad ②$$

First, multiply equation ② by 2 so that the *x* terms have the same coefficient:

$$2x - 3y = -6 \quad ① \qquad 2x + 2y = 4 \quad ②$$

Next, subtract equation ② from equation ①:

$$\begin{array}{r} 2x - 3y = -6 \\ \underline{2x + 2y = 4} \\ -5y = -10 \end{array}$$

Solve for *y*:

$$-5y = -10$$
$$y = 2$$

Example: Elimination Method (cont'd)

Solve the following pair of equations.

$$2x - 3y = -6 \quad ① \qquad x + y = 2 \quad ②$$

Solve for *y*:

$$-5y = -10$$
$$y = 2$$

Substitute $y = 2$ back into equation ① to solve for x :

$$2x - 3(2) = -6$$
$$2x = -6 + 6$$
$$x = 0$$

So, the solution is $(x, y) = (0,2)$

Simple Interest

Learning Objectives

LO1 Calculate interest, maturity value, present value, rate and time in a simple interest environment.

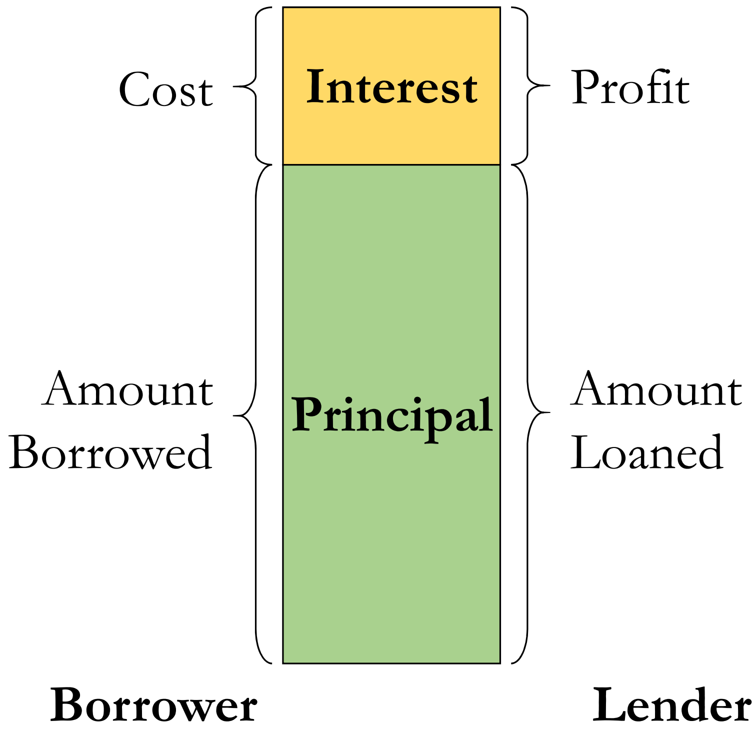
LO2 Present details of the amount and timing of payments in a time diagram.

LO3 Calculate the equivalent value on any date of a single payment or stream of payments.

Introduction

- Debt plays an important role in our personal and business lives.
- Interest is the fee that lenders charge to borrowers for the use of their money.
- Simple interest is used for short-terms loans and investments.
- We will learn how to calculate simple interest and explore the time value of money.

LO1: Simple Interest Basics



P = principal

I = simple interest

r = the interest rate

t = time

Generally, we express the interest rate and time in years.

LO1: Simple Interest Basics

- The formula for the amount of simple interest is:

$$I = Prt$$

P = principal
I = simple interest
r = the interest rate
t = time
Generally, we express the interest rate and time in years.

Example: Calculating the Interest Amount

Suppose that you borrow $500 for one year at a rate of 6% per annum. How much simple interest will you pay?

Given: $P = \$500 \quad t = 1 \quad r = 0.06 \quad I = ?$

The amount of interest you will pay is:

$$I = Prt = \$500(0.06)(1) = \$30.00$$

*Note that the interest rate is converted to decimal form.

The Formula for Simple Interest

- We can re-arrange the formula to solve for P, r or t.

$$P = \frac{I}{rt} \qquad r = \frac{I}{Pt} \qquad t = \frac{I}{Pr}$$

Variations of the Formula

- We can re-arrange the basic simple interest formula to give us three new versions:

$$P = \frac{I}{Pr} \qquad r = \frac{I}{Pt} \qquad t = \frac{I}{Pr}$$

Example: Calculating the Principal Amount

If a 3 month term deposit pays a simple interest rate of 1.5%, how much must be deposited in order to earn $100 in interest?

Given: $t = \frac{3}{12}$ $r = 0.015$ $I = \$100$ $P = ?$

The amount to be deposited is:

$$P = \frac{I}{rt} = \frac{\$100}{0.015\left(\frac{3}{12}\right)} = \$26,666.67$$

Example: Calculating the Interest Rate

Interest of $78.42 was charged on a loan of $1500 for seven months. What simple annual rate of interest was charged on the loan?

Given: $\text{I} = \$78.42 \quad \text{P} = \$1500 \quad t = \frac{7}{12} \quad r = ?$

The rate of simple interest was:

$$r = \frac{I}{Pt} = \frac{\$78.42}{\$1500\left(\frac{7}{12}\right)} = 0.0896 = 8.96\%$$

Example: Calculating the Time Period

How long must you invest $4,000 at a simple interest rate of 2.5% in order to earn $75 interest?

Given: $\mathrm{P} = \$4000 \quad \mathrm{r} = 0.025 \quad \mathrm{I} = \$75 \quad t = ?$

The amount of time needed to earn $75 is:

$$t = \frac{I}{Pr} = \frac{\$75}{\$4000(0.025)} = 0.75 \text{ years or } 9 \text{ months}$$

Determining the Time Period

- If the problem is given in months, you can easily convert to years (i.e. 3 months =3/12 years).
- When using dates, however, it is usually best to work with the exact number of days.
- To do this, you can use Table 6.2 or you can use the date function on your calculator (appendix 6B shows this for the TI calculator).
- Once you have the number of days, you can convert to years by dividing by 365 (exact method).

TABLE 6.2
The Serial Numbers for Each Day of the Year

Day of Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Day of Month
1	1	32	60	91	121	152	182	213	244	274	305	335	1
2	2	33	61	92	122	153	183	214	245	275	306	336	2
3	3	34	62	93	123	154	184	215	246	276	307	337	3
4	4	35	63	94	124	155	185	216	247	277	308	338	4
5	5	36	64	95	125	156	186	217	248	278	309	339	5
6	6	37	65	96	126	157	187	218	249	279	310	340	6
7	7	38	66	97	127	158	188	219	250	280	311	341	7
8	8	39	67	98	128	159	189	220	251	281	312	342	8
9	9	40	68	99	129	160	190	221	252	282	313	343	9
10	10	41	69	100	130	161	191	222	253	283	314	344	10
11	11	42	70	101	131	162	192	223	254	284	315	345	11
12	12	43	71	102	132	163	193	224	255	285	316	346	12
13	13	44	72	103	133	164	194	225	256	286	317	347	13
14	14	45	73	104	134	165	195	226	257	287	318	348	14
15	15	46	74	105	135	166	196	227	258	288	319	349	15
16	16	47	75	106	136	167	197	228	259	289	320	350	16
17	17	48	76	107	137	168	198	229	260	290	321	351	17
18	18	49	77	108	138	169	199	230	261	291	322	352	18
19	19	50	78	109	139	170	200	231	262	292	323	353	19
20	20	51	79	110	140	171	201	232	263	293	324	354	20
21	21	52	80	111	141	172	202	233	264	294	325	355	21
22	22	53	81	112	142	173	203	234	265	295	326	356	22
23	23	54	82	113	143	174	204	235	266	296	327	357	23
24	24	55	83	114	144	175	205	236	267	297	328	358	24
25	25	56	84	115	145	176	206	237	268	298	329	359	25
26	26	57	85	116	146	177	207	238	269	299	330	360	26
27	27	58	86	117	147	178	208	239	270	300	331	361	27
28	28	59	87	118	148	179	209	240	271	301	332	362	28
29	29	*	88	119	149	180	210	241	272	302	333	363	29
30	30		89	120	150	181	211	242	273	303	334	364	30
31	31		90		151		212	243		304		365	31

Determining the Time Period

- Let's use table 6.2 to calculate the number of days between March 3rd and August 21st.

- March 3rd is day 62 and August 21st is day 233.

- The number of days between these two dates is:

$$\#\text{ days between dates} = 233 - 62 = 171$$

Variable or Floating Interest Rates

- The interest rate on loans is often linked to the prime rate charged by banks.
- The prime rate is the banks lowest lending rate.
- Usually loans charge interest at prime plus anywhere from ½% to 5%.
- The prime rate is adjusted by banks to reflect changes in financial markets, so the interest rate payable on a loan may vary over time.

Example 6.2B – Variable Rate of Interest

Lajos borrows $5,000 on April 7 at prime + 1%. The prime rate was initially 3%. It increased to 3.25% on May 23 and to 3.50% on July 13. What was the amount required to repay the loan on August 2?

To solve, we need to break the time period into separate intervals for each rate. We can then use our formula to calculate the interest amount for each interval before adding them together at the end.

Period	# Days	Rate	Interest Amount
Apr 7 – May 23	46	.0400	25.21
May 23 – Jul 13	51	.0425	29.69
Jul 13 – Aug 2	20	.0450	12.33
			67.23

So, the total to be repaid on August 2nd is $5,067.23

LO1: Maturity Value (Future Value)

- When a loan or investment reaches the end of its term, we say that it matures.
- The maturity value (or future value) is the total of the original principal plus interest due on the maturity date.
- The maturity value is represented by the symbol S.

LO1: Maturity Value (Future Value)

- The basic formula for the maturity value is:

$$S = P + I$$

- We know that I=Prt, so we can re-write the formula as:

$$S = P + Prt$$

- We can then re-arrange terms as follows:

$$S = P(1 + rt)$$

Example: Calculating the Maturity Value

You borrow $2,000 for eight months at a simple interest rate of 5% p.a. What is the maturity value of the loan?

Given: $\mathrm{P} = \$2000 \quad \mathrm{r} = 0.05 \quad t = \frac{8}{12} \quad S = ?$

The maturity value of the loan is:

$$S = P(1 + rt) = \$2000\left(1 + 0.05\left(\frac{8}{12}\right)\right) = \$2{,}066.67$$

LO1: Present Value

- We can re-arrange our maturity value formula to solve for the present value.
- In other words, what original amount must be invested in order to reach a specified maturity?

$$S = P(1 + rt)$$

becomes

$$P = \frac{S}{1 + rt}$$

Example: Calculating the Present Value

What amount of money would have to be invested at a simple interest rate of 3% p.a. in order to have $4,000 after 175 days?

Given: $S = \$4000 \quad r = 0.03 \quad t = \frac{175}{365} \quad P = ?$

The amount needed (present value) is:

$$P = \frac{S}{(1+rt)} = \frac{\$4000}{\left(1 + 0.03\left(\frac{175}{365}\right)\right)} = \$3,943.28$$

Time Value of Money

- Money received at different points in time may have different values.

- For example, $100 today is not the same as $100 in one year's time.
- To find out what today's $100 will be worth in 1 year, we need to add interest at an appropriate rate.

- This is called the time value of money.

Equivalent Payments

- An equivalent payment has the same value as another payment once interest has been applied.
- For example, using a rate of 5%, the equivalent payment in one year to $100 today would be $105.
- Both amounts represent the same value, just at different points in time.
- We can use the maturity value and present value formulas to calculate equivalent payments.

Example: Calculating the Equivalent Value at a Later Date

- Bob had promised to pay Sally $1,000 on Oct 1st. Sally planned on investing the money into a GIC earning 2.5%.
- Bob was late with his payment and couldn't pay Sally until Nov 10th. What would be a fair amount for Bob to pay?

Given: $\mathrm{P} = \$1000 \quad \mathrm{r} = 0.025 \quad t = \frac{40}{365} \quad S = ?$

The equivalent value on Nov 10th is:

$$S = P(1 + rt) = \$1000\left(1 + 0.025\left(\frac{40}{365}\right)\right) = \$1,002.74$$

Example: Calculating the Equivalent Value at an Earlier Date

- Bob had promised to pay Sally $1,000 on Oct 1st but would like to pay her back a few weeks earlier on Sept 10th.
- What would be a fair amount for Bob to pay if Sally can invest his payment at a rate of 2.5%?

Given: $S = \$1000 \quad \mathrm{r} = 0.025 \quad t = \frac{21}{365} \quad P = ?$

The equivalent amount on September 10th is:

$$P = \frac{S}{(1+rt)} = \frac{\$1000}{\left(1 + 0.025\left(\frac{21}{365}\right)\right)} = \$998.56$$

LO3: The Equivalent Value of a Payment Stream

- A payment stream is simply a set of payments over time.
- To get the combined value of a payment stream, you can't just add the payments up.
- You must take the time value of money into account.
- When solving these types of problems, a good timeline is helpful.

LO2: Construct a Time Diagram

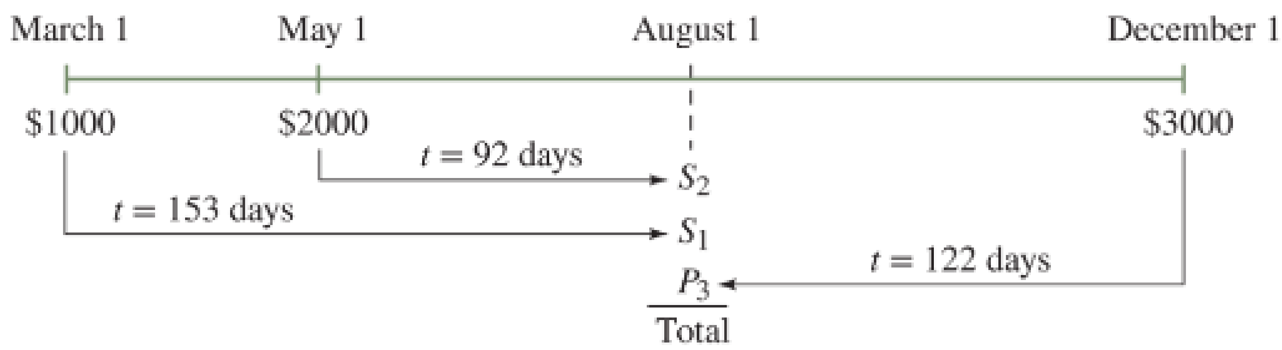
- A timeline shows all of the amounts in the payment stream on the dates that they occur.
- This is an excellent tool for helping us to determine an equivalent value to the payment stream.

- Tip: A clearly drawn timeline can really help.

Example: Equivalent Value to a Payment Stream

Consider a payment stream consisting of three payments: $1000, $2000, and $3000, scheduled for March 1, May 1, and December 1 of the same year. Calculate the single payment on August 1 that is equivalent to the three scheduled payments. Use a simple interest rate of 8%.

Set up a timeline with August first as the focal date. The equivalent value will be the maturity value of the March 1 and May 1 amounts plus the present value of the December 1 amount:



Example: Equivalent Value to a Payment Stream (cont'd)

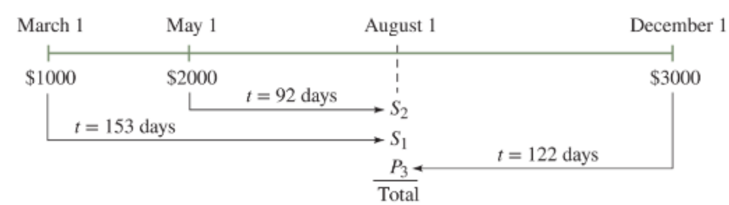
Consider a payment stream consisting of three payments: $1000, $2000, and $3000, scheduled for March 1, May 1, and December 1 of the same year. Calculate the single payment on August 1 that is equivalent to the three scheduled payments. Use a simple interest rate of 8%.

Calculate the individual equivalent values at the focal date and then add them:

$$\begin{aligned} S_1 &= \text{Future value on August 1 of the \$1000 payment} \\ &= \$1000\left(1+0.08\times\frac{153}{365}\right) \\ &= \$1033.53 \end{aligned}$$

$$\begin{aligned} S_2 &= \text{Future value on August 1 of the \$2000 payment} \\ &= \$2000\left(1+0.08\times\frac{92}{365}\right) \\ &= \$2040.33 \end{aligned}$$

$$\begin{aligned} P_3 &= \text{Present value on August 1 of the \$3000 payment} \\ &= \frac{\$3000}{1+0.08\times\frac{122}{365}} \\ &= \$2921.87 \end{aligned}$$



So, the total equivalent value on August 1st will be $5,995.73

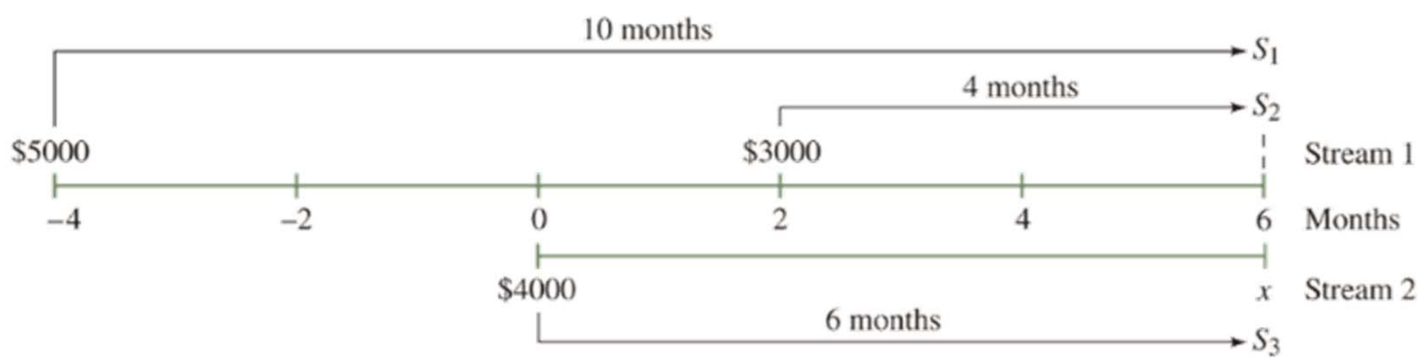
Be Careful!

- Some people are tempted to simply move the first amount forward to the second, add them, then move the new total forward to the third, etc.
- This will result in the wrong answer, however, as it compounds the interest.
- Remember these rules:
    1. Payments can be moved only once (to the focal date).
    2. Payments may only be added once they are at the focal date.
    3. Use only one focal date.

Example 6.5C: Calculating an Unknown Payment in an Equivalent Payment Stream

Payments of $5000 due four months ago and $3000 due two months from now are to be replaced by a payment of $4000 today and a second payment in six months. What must the second payment be in order to make the replacement payment stream equivalent to the scheduled payment stream? Money in short-term investments can earn 5%. Use six months from now as the focal date.

Set up a timeline with one payment stream on the top and one on the bottom. Set the focal date at 6 months and use *x* to represent the unknown payment:



Example 6.5C: Calculating an Unknown Payment in an Equivalent Payment Stream (cont'd)

Payments of $5000 due four months ago and $3000 due two months from now are to be replaced by a payment of $4000 today and a second payment in six months. What must the second payment be in order to make the replacement payment stream equivalent to the scheduled payment stream? Money in short-term investments can earn 5%. Use six months from now as the focal date.

Calculate the equivalent value of each stream and set them equal to each other to solve for the unknown value (*x*):

$$\begin{aligned}
\text{Equivalent value of Stream 1} &= S_1 + S_2 \\
&= \$5000\left(1 + 0.05 \times \frac{10}{12}\right) + \$3000\left(1 + 0.05 \times \frac{4}{12}\right) \\
&= \$5208.333 + \$3050.000 \\
&= \$8258.333 \\
\text{Equivalent value of Stream 2} &= x + S_3 \\
&= x + \$4000\left(1 + 0.05 \times \frac{6}{12}\right) \\
&= x + \$4100.0000
\end{aligned}$$

For the two streams to be economically equivalent,

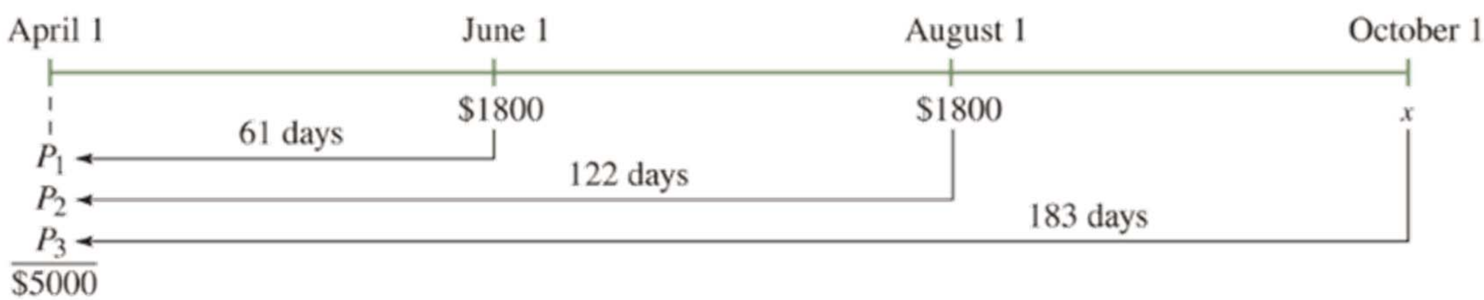
$$x + \$4100.000 = \$8258.333$$

Hence, $x = \$4158.33$

Example 6.6A: Calculating the Size of the Final Loan Payment

A $5000 loan advanced on April 1 at a 6.5% interest rate requires payments of $1800 on each of June 1 and August 1, and a final payment on October 1. What must the final payment be to satisfy the loan in full?

Let *x* represent the final payment. Set up a timeline with the focal date at the original loan date (April 1). The present value of all three payments added together should be equal to the $5,000 loan amount:



Example 6.6A: Calculating the Size of the Final Loan Payment

A $5000 loan advanced on April 1 at a 6.5% interest rate requires payments of $1800 on each of June 1 and August 1, and a final payment on October 1. What must the final payment be to satisfy the loan in full?

$$P_1 = \frac{\$1800}{1 + 0.065\left(\frac{61}{365}\right)} = \frac{\$1800}{1.0108630} = \$1780.657$$

$$P_2 = \frac{\$1800}{1 + 0.065\left(\frac{122}{365}\right)} = \frac{\$1800}{1.0217260} = \$1761.725$$

$$P_3 = \frac{x}{1 + 0.065\left(\frac{183}{365}\right)} = \frac{x}{1.0325890} = 0.9684395x$$

We maintain seven-figure precision in order to ensure six-figure accuracy in the final result.

Thus

$$\$5000 = \$1780.657 + \$1761.725 + 0.9684395x$$

$$\$1457.619 = 0.9684395x$$

$$x = \frac{\$1457.619}{0.9684395} = \$1505.12$$

The final payment on October 1 must be $1505.12.

End of Chapter 1

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